FLUXBRANE AND S-BRANE SOLUTIONS WITH POLYNOMIALS RELATED TO RANK-2 LIE ALGEBRAS

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Composite fluxbrane and S-brane solutions for a wide class of intersection rules are considered. These solutions are defined on a product manifold $R_* \times M_1 \times \ldots \times M_n$ which contains n Ricci-flat spaces $M_1, \ldots, M_n$ with 1-dimensional factor spaces R_* and M_1. They are determined up to a set of functions obeying non-linear differential equations equivalent to Toda-type equations with certain boundary conditions imposed. Exact solutions corresponding to configurations with two branes and intersections related to simple Lie algebras C_2 and G_2 are obtained. In these cases, the functions $H_s(z)$, $s = 1, 2$, are polynomials of degrees $(3,\ 4)$ and $(6,\ 10)$, respectively, in agreement with a conjecture put forward previously in Ref. [1]. The S-brane solutions under consideration, for special choices of the parameters, may describe an accelerating expansion of our 3-dimensional space and a small enough variation of the effective gravitational constant.

1. Introduction

In this paper, we deal with the so-called multidimensional fluxbrane solutions (see [1, 3–18] and references therein) that are in fact generalizations of the well-known Melvin solution [2]. (Melvin's original solution describes the gravitational field of a magnetic flux tube.)

In [1], a subclass of generalized fluxbrane solutions was obtained. These fluxbrane solutions are governed by functions $H_s(z) > 0$ defined on the interval $(0, +\infty)$ and obeying a certain set of second-order nonlinear differential equations,

$$\frac{d}{dz}\left(\frac{z}{H_s}\frac{d}{dz}H_s\right) = \frac{1}{4}B_s \prod_{s' \in S} H_{s'}^{-A_{ss'}}, \tag{1.1}$$

with the boundary conditions

$$H_s(+0) = 1, \tag{1.2}$$

$s \in S$ (S is a non-empty set). In (1.1), all $B_s \neq 0$ are constants and $(A_{ss'})$ is the so-called "quasi-Cartan" matrix ($A_{ss} = 2$) coinciding with the Cartan matrix when intersections are related to Lie algebras.

In [1], the following **hypothesis** was suggested: the solutions to Eqs. (1.1), (1.2) (if they exist) are polynomials when the intersection rules correspond to semisimple Lie algebras. As pointed in [1], this hypothesis could be readily verified for the Lie algebras A_m, C_{m+1}, $m = 1, 2, \ldots$, as was done for black-brane solutions from [22, 23].

In [1], explicit formulae for solutions corresponding to the Lie algebras $A_1 \oplus \ldots \oplus A_1$ and A_2 were presented.

In this paper, we consider generalized "flux-S-brane" solutions depending on the parameter $w = \pm 1$. For $w = +1$, these solutions are coinciding with flux-brane solutions from [1]. For $w = -1$, they describe special S-brane solutions with Ricci-flat factor spaces. (For general S-brane configurations see [19] and references therein.) Here we present new solutions with polynomials $H_s(z)$ related to the algebras C_2 and G_2.

The S-brane solutions corresponding to the Lie algebras A_2, C_2 and G_2, for special choices of the parameters, may describe an accelerating expansion of "our" 3-dimensional space with small enough variations of the effective gravitational constant [24] (for the case of the A_2 algebra see also [25]).

2. Flux- and S-brane solutions with general intersection rules

2.1. The model

We consider a model governed by the action

$$S = \int d^D x \sqrt{|g|}\Big\{R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{\theta_a}{N_a!} \exp[2\lambda_a(\varphi)](F^a)^2\Big\}, \tag{2.1}$$

where $g = g_{MN}(x) dx^M \otimes dx^N$ is the metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of scalar fields, $(h_{\alpha\beta})$ is a constant symmetric non-degenerate $l \times l$ matrix ($l \in \mathbb{N}$), $\theta_a = \pm 1$,

$$F^a = dA^a = \frac{1}{N_a!} F^a_{M_1 \ldots M_{n_a}} dz^{M_1} \wedge \ldots \wedge dz^{M_{n_a}} \tag{2.2}$$

is an N_a-form ($n_a \geq 1$), λ_a is a 1-form on $\mathbb{R}^l$: $\lambda_a(\varphi) = \lambda_{a\alpha}\varphi^\alpha$, $a \in \Delta$, $\alpha = 1, \ldots, l$. In (2.1), we denote $|g| = |\det(g_{MN})|$,

$$(F^a)^2_g = F^a_{M_1 \ldots M_{n_a}} F^a_{N_1 \ldots N_{n_a}} g^{M_1 N_1} \ldots g^{M_{n_a} N_{n_a}}, \tag{2.3}$$

$a \in \Delta$, where Δ is some finite set.

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2.2. "Flux-S-brane" solutions

Let us consider a family of exact solutions to the field equations corresponding to the action (2.1) and depending on one variable ρ. These solutions are defined on the manifold

$$M = (0, +\infty) \times M_1 \times M_2 \times \ldots \times M_n, \tag{2.4}$$

where M_1 is a one-dimensional manifold. The solutions read

$$g = \Big(\prod_{s \in S} H_s^{2h_s d(I_s)/(D-2)}\Big)\Big\{ w d\rho \otimes d\rho + \Big(\prod_{s \in S} H_s^{-2h_s}\Big)\rho^2 g^1 + \sum_{i=2}^{n}\Big(\prod_{s \in S} H_s^{-2h_s\delta_{iI_s}}\Big) g^i \Big\}, \tag{2.5}$$

$$\exp(\varphi^\alpha) = \prod_{s \in S} H_s^{h_s \chi_s \lambda_{a_s}^\alpha}, \tag{2.6}$$

$$F^a = \sum_{s \in S} \delta^a_{a_s} \mathcal{F}^s, \tag{2.7}$$

where

$$\mathcal{F}^s = -Q_s \left(\prod_{s' \in S} H_{s'}^{-A_{ss'}}\right) \rho d\rho \wedge \tau(I_s), \quad s \in S_e, \tag{2.8}$$

$$\mathcal{F}^s = Q_s \tau(\bar{I}_s), \qquad s \in S_m. \tag{2.9}$$

The functions $H_s(z) > 0$, $z = \rho^2$ obey Eqs. (1.1) with the boundary conditions (1.2).

In (2.5), $g^i = g^i_{m_i n_i}(y_i) dy_i^{m_i} \otimes dy_i^{n_i}$ is a Ricci-flat metric on M_i, $i = 1, \ldots, n$,

$$\delta_{iI} = \sum_{j \in I} \delta_{ij} \tag{2.10}$$

is the indicator of i belonging to I: $\delta_{iI} = 1$ for $i \in I$ and $\delta_{iI} = 0$ otherwise.

The brane set S is, by definition, a union of two sets:

$$S = S_e \cup S_m, \quad S_v = \cup_{a \in \Delta}\{a\} \times \{v\} \times \Omega_{a,v}, \tag{2.11}$$

$v = e, m$ and $\Omega_{a,e}, \Omega_{a,m} \subset \Omega$, where $\Omega = \Omega(n)$ is the set of all non-empty subsets of $\{1, \ldots, n\}$. Any brane index $s \in S$ has the form

$$s = (a_s, v_s, I_s), \tag{2.12}$$

where $a_s \in \Delta$ is the colour index, $v_s = e, m$ is the electro-magnetic index, and the set $I_s \in \Omega_{a_s, v_s}$ describes the location of the brane worldvolume.

The sets S_e and S_m define electric and magnetic branes, respectively. In (2.6),

$$\chi_s = +1, -1 \tag{2.13}$$

for $s \in S_e$, S_m, respectively. In (2.7), the forms (2.8) correspond to electric branes and the forms (2.9) to magnetic branes; $Q_s \neq 0$, $s \in S$. In (2.9) and in what follows,

$$\bar{I} \equiv I_0 \setminus I, \qquad I_0 = \{1, \ldots, n\}. \tag{2.14}$$

All manifolds M_i are assumed to be oriented and connected, and the volume d_i-forms

$$\tau_i \equiv \sqrt{|g^i(y_i)|}\, dy_i^1 \wedge \ldots \wedge dy_i^{d_i}, \tag{2.15}$$

and the parameters

$$\varepsilon(i) \equiv \operatorname{sign}\det(g^i_{m_i n_i}) = \pm 1 \tag{2.16}$$

are well defined for all $i = 1, \ldots, n$. Here $d_i = \dim M_i$, $i = 1, \ldots, n$, $D = 1 + \sum_{i=1}^n d_i$. For any $I = \{i_1, \ldots, i_k\} \in \Omega$, $i_1 < \ldots < i_k$, we denote

$$\tau(I) \equiv \tau_{i_1} \wedge \ldots \wedge \tau_{i_k}, \tag{2.17}$$

$$M(I) \equiv M_{i_1} \times \ldots \times M_{i_k}, \tag{2.18}$$

$$d(I) \equiv \dim M(I) = \sum_{i \in I} d_i, \tag{2.19}$$

$$\varepsilon(I) \equiv \varepsilon(i_1) \ldots \varepsilon(i_k). \tag{2.20}$$

$M(I_s)$ is isomorphic to a brane worldvolume (see (2.12)).

The parameters h_s appearing in the solution satisfy the relations

$$h_s = K_s^{-1}, \qquad K_s = B_{ss}, \tag{2.21}$$

where

$$B_{ss'} \equiv d(I_s \cap I_{s'}) + \frac{d(I_s) d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{a_s \alpha} \lambda_{a_{s'} \beta} h^{\alpha\beta}, \tag{2.22}$$

$s, s' \in S$, with $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$. In (2.6), $\lambda^\alpha_{a_s} = h^{\alpha\beta}\lambda_{a_s \beta}$.

We will assume that

$$\textbf{(i)} \qquad B_{ss} \neq 0, \tag{2.23}$$

for all $s \in S$, and

$$\textbf{(ii)} \qquad \det(B_{ss'}) \neq 0, \tag{2.24}$$

i.e. the matrix $(B_{ss'})$ is nondegenerate. In (2.8), there appears another nondegenerate matrix (the so-called "quasi-Cartan" matrix)

$$(A_{ss'}) = (2 B_{ss'} / B_{s's'}) \,. \tag{2.25}$$

In (1.1),

$$B_s = \varepsilon_s K_s Q_s^2, \qquad s \in S, \tag{2.26}$$

where

$$\varepsilon_s = (-\varepsilon[g])^{(1-\chi_s)/2} \varepsilon(I_s) \theta_{a_s}, \tag{2.27}$$

$s \in S$, $\varepsilon[g] \equiv \operatorname{sign}\det(g_{MN})$.

More explicitly, (2.27) reads: $\varepsilon_s = \varepsilon(I_s)\theta_{a_s}$ for $v_s = e$ and $\varepsilon_s = -\varepsilon[g]\varepsilon(I_s)\theta_{a_s}$, for $v_s = m$.

Due to (2.8) and (2.9), the brane worldvolume dimension $d(I_s)$ is determined as

$$d(I_s) = N_{a_s} - 1, \qquad d(I_s) = D - N_{a_s} - 1, \tag{2.28}$$

for $s \in S_e$, S_m, respectively. For an Sp-brane: $p = p_s = d(I_s) - 1$.

Restrictions on brane configurations. The solutions presented above are valid if two restrictions on the sets of branes are satisfied. These restrictions guarantee a block-diagonal form of the energy-momentum tensor and the existence of the sigma-model representation (without additional constraints) [27, 20]. These restrictions are:

$$(\mathbf{R1}) \qquad d(I \cap J) \leq d(I) - 2 \tag{2.29}$$

for any $I, J \in \Omega_{a,v}$, $a \in \Delta$, $v = e, m$ (and here $d(I) = d(J)$).

$$(\mathbf{R2}) \qquad d(I \cap J) \neq 0, \tag{2.30}$$

for $I \in \Omega_{a,e}$ and $J \in \Omega_{a,m}$, $a \in \Delta$.

In the cylindrically symmetric case,

$$M_1 = S^1, \qquad g^1 = d\phi \otimes d\phi, \tag{2.31}$$

$0 < \phi < 2\pi$ and $w = +1$, we get the family of composite fluxbrane solutions from [1].

3. Conjecture on a polynomial structure of H_s for Lie algebras

In what follows, we consider the case

$$\varepsilon_s > 0, \tag{3.1}$$

$$K_s > 0. \tag{3.2}$$

In this case all $B_s > 0$. Eq. (3.1) is satisfied when all $\theta_a > 0$, $\varepsilon[g] = -1$ (e.g., when the metric g has a pseudo-Euclidean signature $(-, +, ..., +)$) and

$$\varepsilon(I_s) = +1 \tag{3.3}$$

for all $s \in S$. (The relation (3.3) takes place for S-brane and fluxbrane solutions.) The second relation (3.2) takes place when all $d(I_s) < D - 2$ and the matrix $(h_{\alpha\beta})$ is positive-definite (i.e., there are no phantom scalar fields).

Let us consider the second-order differential equations (1.1) with the boundary conditions (1.2) for the functions $H_s(z) > 0$, $s \in S$. We will be interested in analytical solutions to Eqs. (1.1) in some disc $|z| < L$:

$$H_s(z) = 1 + \sum_{k=1}^{\infty} P_s^{(k)} z^k, \tag{3.4}$$

where $P_s^{(k)}$ are constants, $s \in S$. Substitution of (3.4) into (1.1) gives an infinite chain of relations for the parameters $P_s^{(k)}$ and B_s. The first relation in this chain

$$P_s \equiv P_s^{(1)} = \frac{1}{4} B_s = \frac{1}{4} K_s Q_s^2, \tag{3.5}$$

$s \in S$, corresponds to the z^0-term in the decomposition of (1.1).

It can by shown that, for analytic functions $H_s(z)$, $s \in S$ (3.4) ($z = \rho^2$), the metric (2.5) is regular at $\rho = 0$ for $w = +1$, i.e. in the fluxbrane case.

Let $(A_{ss'})$ be a Cartan matrix of a finite-dimensional semisimple Lie algebra $\mathcal{G}$.

It has been conjectured in [1] that there exist polynomial solutions to Eqs. (1.1), (1.2), having the form

$$H_s(z) = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k, \tag{3.6}$$

where $P_s^{(k)}$ are constants, $k = 1, \ldots, n_s$. Here, $P_s^{(n_s)} \neq 0$, $s \in S$, and

$$n_s = 2 \sum_{s' \in S} A^{ss'}. \tag{3.7}$$

The integers n_s are components of the so-called twice dual Weyl vector in the basis of simple roots [26].

3.1. Examples of solutions for rank-2 Lie algebras

Consider configurations with two branes, i.e., $S = \{s_1, s_2\}$.

3.1.1. Solutions in the $A_1 \oplus A_1$ case

The simplest example occurs in so-called "orthogonal" case, when $(A_{ss'}) = \mathrm{diag}(2, 2)$ is the Cartan matrix of the semisimple Lie algebra $A_1 \oplus A_1$, where $A_1 = sl(2)$. We get [1]

$$H_s(z) = 1 + P_s z, \tag{3.8}$$

with $P_s \neq 0$ satisfying (3.5).

3.1.2. Solutions in the A_2 case

For the Lie algebra $A_2 = sl(3)$ with the Cartan matrix

$$(A_{ss'}) = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, \tag{3.9}$$

we have [1] $n_1 = n_2 = 2$:

$$H_s = 1 + P_s z + P_s^{(2)} z^2, \tag{3.10}$$

where, here and in what follows, P_s obey Eq. (3.5), and

$$P_s^{(2)} = \frac{1}{4} P_1 P_2. \tag{3.11}$$

3.1.3. Solutions for the Lie algebra C_2

For the Lie algebra $C_2 = so(5)$ with the Cartan matrix

$$(A_{ss'}) = \begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix}, \tag{3.12}$$

we get from (3.7): $n_1 = 3$ and $n_2 = 4$. For the moduli functions we obtain

$$H_1 = 1 + P_1 z + \frac{1}{4} P_1 P_2 z^2 + \frac{1}{36} P_1^2 P_2 z^3, \tag{3.13}$$

$$H_2 = 1 + P_2 z + \frac{1}{2} P_1 P_2 z^2 + \frac{1}{9} P_1^2 P_2 z^3 + \frac{1}{144} P_1^2 P_2^2 z^4. \tag{3.14}$$

3.1.4. Solutions for the Lie algebra G_2

Consider now the exceptional Lie algebra G_2 with the Cartan matrix

$$(A_{ss'}) = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}. \tag{3.15}$$

We get from (3.7) $n_1 = 6$ and $n_2 = 10$. Calculations (using MATHEMATICA and MAPLE) give:

$$\begin{aligned} H_1 \; &= \; 1 + P_1 z + \frac{1}{4} P_1 P_2 z^2 + \frac{1}{18} P_1^2 P_2 z^3 + \frac{1}{144} P_1^3 P_2 z^4 \\ &+ \frac{1}{3600} P_1^3 P_2^2 z^5 + \frac{1}{129600} P_1^4 P_2^2 z^6, \end{aligned} \tag{3.16}$$

$$\begin{aligned} H_2 \; &= \; 1 + P_2 z + \frac{3}{4} P_1 P_2 z^2 + \frac{1}{3} P_1^2 P_2 z^3 \\ &+ \frac{1}{16} P_1^2 P_2 (\frac{1}{3} P_2 + P_1) z^4 + \frac{7}{600} P_1^3 P_2^2 z^5 \\ &+ \frac{1}{64} P_1^3 P_2^2 \left(\frac{1}{25} P_2 + \frac{1}{27} P_1 \right) z^6 + \frac{1}{10800} P_1^4 P_2^3 z^7 \\ &+ \frac{1}{172800} P_1^5 P_2^3 z^8 + \frac{1}{4665600} P_1^6 P_2^3 z^9 \\ &+ \frac{1}{466560000} P_1^6 P_2^4 z^{10}. \end{aligned} \tag{3.17}$$

The intersection rules are given by Eqs. (2.25).

We would like to outline some useful relations for the big numbers appearing in the denominators of the polynomial coefficients: $1296 = 6^4$, $1728 = 3(24^2)$, $46656 = 6^6$.

There are at least two ways of calculating the coefficients $P_s^{(k)}$ of the polynomials. The first one (performed by MAPLE) consists in a straightforward substitution of the polynomials (3.6) into the "master" equations (1.1). The second one (carried out with MATHEMATICA) uses recurrent relations for the coefficients $P_s^{(k+1)}$ as functions of other coefficients $P_s^{(1)}, \ldots, P_s^{(k)}$. These recurrent relations were obtained analytically by simply decomposing the "master" equations (1.1) into a power series in the parameters z [28].

4. Conclusions

We have presented explicit formulae for fluxbrane and S-brane solutions governed by polynomials which correspond to Lie algebras: $A_1 \oplus A_1$, A_2, C_2 and G_2. The pairs of moduli functions (H_1, H_2) in these solutions are polynomials of degrees: (1, 1), (2, 2), (3, 4) and (6, 10), in agreement with a conjecture from Ref. [1]. The general S-brane solutions presented here, governed by polynomials, are new. The fluxbrane solutions related to Lie algebras C_2 and G_2 are new as well.

Acknowledgement

This work was supported in part by the Russian Foundation for Basic Research grant Nr. 05-02-17478 and by DFG grant Nr. 436 RUS 113/807/0-1.

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